# HISSAN RUPANDEHI

## Pre-Board Examination 2078 (2022)

**Grade: XII**

**Mathematics, MORNING**

*Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.*

**Time: 3:00 Hours** **Full Marks: 75**

### Group "A"

**Rewrite the correct option in your answer sheet.**

1. If $C(18,r) = C(18,r+2)$, then the value of 'r' is

   a) 2 b) 4 c) 8 d) 18

2. If the roots of the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, then a,b,c are in

   a) A.P. b) G.P c) HP d) None

3. If the augmented matrix of system of linear equations reduces to $\begin{bmatrix} 1 & 0 & 0: & 1 \\ 0 & 1 & 0: & 2 \\ 0 & 0 & 0: & 5 \end{bmatrix}$ then the system has

   a) No solution b) One solution

   c) Two solution d) Infinitely many solutions

4. The general solution of $7\,Sin^2x + 3Cos^2x = 4$ is

   a) $n\pi \pm \frac{\pi}{6}$ b) $n\pi \pm \frac{2\pi}{3}$ c) $n\pi \pm \frac{\pi}{4}$ d) $2n\pi$

5. The parametric equation $x = a\cos\theta$ and $y = b\,Sin\theta$ represents

   a) Circle b) Ellipse c) Parabola d) Hyperbola

6. The probability of a sure event is

   a) 0 b) 1 c) $\frac{1}{2}$ d) $\frac{3}{2}$

7. The derivative of $tanh^{-1}(sinx)$ with respect to $x$ is

   a) $Cos\,x$ b) $Sec\,x$ c) $Sec\,hx$ d) $tanx$

8. The value of $\int \frac{1}{e^x+e^{-x}}dx$ is

   a) $tan^{-1}(e^{-x}) + c$ b) $tan^{-1}(e^x) + c$

   c) $-tan^{-1}(e^x) + c$ d) $-tan^{-1}(e^{-x}) + c$

16. Answer the following questions:
   a) Write a difference between derivative and anti-derivative. [1]
   b) What is the slope and inclination with the x-axis of the tangent of $y=-3x-x^4$ at $x=-1$ [2]
   c) Integrate: $\int \frac{dx}{x^2+12x+45}$ [2]

17. Solve: $(1+x^2)\frac{dy}{dx} + y = e^{tan^{-1}x}$ [5]

18. Solve the following LP problem, using simplex method.
   Maximize $P = 30x + 20y$ Subject to
   $2x + y \leq 24$
   $x + 2y \leq 15$
   $x, y \geq 0$

19. Write any one difference between like parallel forces and unlike parallel forces. A heavy uniform beam whose mass is 60kg is suspended in a horizontal position by two vertical strings each of which can sustain a tension of 52.5kg wt. How far from the centre of the beam must a body of mass 30kg placed so that one of the strings may just break? [1+4=5]

**OR**

In a perfect competition, the demand and supply curves for a commodity are given by $D(x) = 40 - x^2$ and $S(x) = 3x^2 + 8x + 8$. Find the consumer surplus and prouder surplus at the market equilibrium price.[5]

## Group "C"

**Attempt all the questions:** **[8×3=24]**

20. a) State Binomial theorem. In the expansion of $(1+x)^n$ prove that the sum of the coefficients of the odd terms is equal to the sum of the coefficients of the even terms and each equal to $2^{n-1}$. [4]
   b) State De-Moivers theorem. Using this theorem find the square roots of $4 + 4\sqrt{3}\,i$ [4]

21. a) If $Sin^{-1}x + Sin^{-1}y + Sin^{-1}z = \frac{\pi}{2}$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$ [3]

9. The order of the differential equation $3\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^4 = sinx$ is

a) 1 b) 2 c) 3 d) 4

10. In Gauss eliminating method, the coefficient of the first variable in first equation must be

a) Zero b) Non-zero c) Negative d) Positive

11. When do we get maximum range in a simple projectile motion?

a) $\alpha = 0^0$ b) $\alpha = 30^0$ c) $\alpha = 45^0$ d) $\alpha = 90^0$

**OR**

The producer surplus of the supply function $x = -8 + 2P$ at $x = 4$ is

a) 1 b) 4 c) 3 d) 6

## Group "B"

12. a) In how many ways the letters of the word "CALCULUS" can be arranged so that vowels are always together. [2]

b) Find the eccentricity, foci and latus rectum of the hyperbola: $3x^2 - 4y^2 = 36$ [3]

13. a) For a certain bivariate data

| | x | y |
|---|---|---|
| Mean | 10 | 18 |
| SD | 2.5 | 2.0 |

And the coefficient of correlation between 'x' and 'y' is 0.8 determine the following:

i) The regression equations of 'y' on 'x' and regression of 'x' on 'y'

ii) Estimated value of 'y' for x=15

b) A coin is tossed successively three times. Determine the probability of getting 2 heads and one tail. [2]

14. Find the direction cosines of two lines satisfy the equations $l + m + n = 0$ and $2lm + 2ln - mn = 0$ Also find the angle between them. [5]

15. a) Let G={0,1,2} form a composition table for 'G' under multiplication module 3. Find the identify element of 2. [2]

b) Prove by induction method that $2^{3n}$-1 is divisible by 7. [3]

b) Define vector product of two vectors interpret it geometrically and hence find the area of the parallelogram whose adjacent sides are the vectors. $\vec{u} = 2\vec{i} + 3\vec{j}$ *and* $\vec{v} = \vec{i} + 4\vec{j}$ [1+2+2]

22. a) State Rolle's theorem. interpret it geometrically. Verify that the function $f(x) = \sin x + \cos x$ on $[0, 2\pi]$ satisfies the conditions of Rolle's theorem and find the constant c prescribed by the theorem. If $f(x) = (x - 1)^2$, Show that $f(0) = f(2)$, but there is no number C in (0,2) such that $f'(c) = 0$ [1+1+4]

b) Evaluate: $\int \frac{e^x}{e^{2x-4}} dx$ [2]

***Best of Hard Labour***